Announcements

• Midterm results
  – Mean: 72%
  – Max: 90%, Min: 54%
Register Allocation and Coalescing

• Introduction
• Abstraction and the Problem
• Algorithm
• Spilling
• Coalescing

Reading: ALSU 8.8.4
Motivation

• **Problem**
  – Allocation of variables (pseudo-registers) to hardware registers in a procedure

• **A very important optimization!**
  – Directly reduces running time
    • (memory access $\rightarrow$ register access)
  – Useful for other optimizations
    • e.g. CSE assumes old values are kept in registers.
Goals

• Find an allocation for all pseudo-registers, if possible.

• If there are not enough registers in the machine, choose registers to spill to memory
Register Assignment Example

A = ...
IF A goto L1

B = ...
= A
D =
= B + D

L1: C = ...
= A
D =
= C + D

• Find an assignment (no spilling) with only 2 registers
  – A and D in one register, B and C in another one

• What assumptions?
  – After assignment, no use of A & (and only one of B and C used)
An Abstraction for Allocation & Assignment

• Intuitively
  – Two pseudo-registers interfere if at some point in the program they cannot both occupy the same register.

• Interference graph: an undirected graph, where
  – nodes = pseudo-registers
  – there is an edge between two nodes if their corresponding pseudo-registers interfere

• What is not represented
  – Extent of the interference between uses of different variables
  – Where in the program is the interference

Interferes many times vs. once
E.g., cold path vs. hot path
Register Allocation and Coloring

• A graph is **n-colorable** if:
  – every node in the graph can be colored with one of the n colors such that two adjacent nodes do not have the same color.

• Assigning n register (without spilling) = Coloring with n colors
  – assign a node to a register (color) such that no two adjacent nodes are assigned same registers (colors)

• Is spilling necessary? = Is the graph n-colorable?

• To determine if a graph is n-colorable is **NP-complete, for n>2**
  – Too expensive
  – Heuristics
Algorithm

Step 1. Build an interference graph
   a. refining notion of a node
   b. finding the edges

Step 2. Coloring
   – use heuristics to try to find an n-coloring
     • Success:
       – colorable and we have an assignment
     • Failure:
       – graph not colorable, or
       – graph is colorable, but it is too expensive to color
Step 1a. Nodes in an Interference Graph

A = ...
IF A goto L1

B = ...
= A
D =
= B + D

L1: C = ...
= A
D =
= D + C

A = 2

Should we add A-D edge?
No, since new def of A
Live Ranges and Merged Live Ranges

• Motivation: to create an interference graph that is easier to color
  – Eliminate interference in a variable’s “dead” zones.
  – Increase flexibility in allocation:
    • can allocate same variable to different registers
• A **live range** consists of a definition and all the points in a program in which that definition is live.
  – How to compute a live range?
• Two overlapping live ranges for the **same** variable must be merged
Example (Revisited)

Live Variables

Reaching Definitions

\begin{align*}
A &= \ldots \ (A_1) \\
&\text{IF } A \text{ goto L1} \\
&\{\} \quad \{\} \\
&\{A\} \quad \{A_1\} \\
&\{A\} \quad \{A_1\}
\end{align*}

\begin{align*}
B &= \ldots \ (B_1) \\
&= A \\
&\{A\} \quad \{A_1\} \\
D &= B \ (D_2) \\
&\{A_1, B_1, D_2\}
\end{align*}

\begin{align*}
L1: \\
C &= \ldots \ (C_1) \\
&= A \\
D &= \ldots \ (D_1) \\
&\{A_1, C_1, D_1\} \\
&\{A_1, C_1\}
\end{align*}

\begin{align*}
A &= 2 \ (A_2) \\
&\{D\} \quad \{A, D\} \\
&\{A_1, B_1, C_1, D_1, D_2\} \\
&\{A_1, B_1, C_1, D_1, D_2\}
\end{align*}

\begin{align*}
A &= A \ (A_3) \\
&\{D\} \quad \{A, D\} \\
&\{A_2, B_1, C_1, D_1, D_2\} \\
&\{A_2, B_1, C_1, D_1, D_2\}
\end{align*}

\begin{align*}
\text{ret } D \\
&\text{Merge}
\end{align*}
Merging Live Ranges

• **Merging definitions into equivalence classes**
  – Start by putting each definition in a different equivalence class
  – Then, for each point in a program:
    • if (i) variable is live, and (ii) there are multiple reaching definitions for the variable, then:
      – merge the equivalence classes of all such definitions into one equivalence class
    • *(Sound familiar?)*

• From now on, refer to **merged live ranges** simply as **live ranges**
  – merged live ranges are also known as “**webs**”
SSA Revisited: What Happens to $\Phi$ Functions

• Now we see why it is unnecessary to “implement” a $\Phi$ function
  – $\Phi$ functions and SSA variable renaming simply turn into merged live ranges

• When you encounter: $x_4 = \Phi(x_1, x_2, x_3)$
  – merge $x_1, x_2, x_3$, and $x_4$ into the same live range
  – delete the $\Phi$ function

• Now you have effectively converted back out of SSA form
Step 1b. Edges of Interference Graph

• Intuitively:
  – Two live ranges (necessarily of different variables) may interfere if they overlap at some point in the program.
  – Algorithm:
    • At each point in the program:
      – enter an edge for every pair of live ranges at that point.

• An optimized definition & algorithm for edges:
  – Algorithm:
    • check for interference only at the start of each live range
  – Faster
  – Better quality
Live Range Example 2

Because ranges overlap: Won’t assign A and B to same register (even though would have been ok: path sensitive vs. path insensitive analysis)
Step 2. Coloring

• Reminder: **coloring for n > 2 is NP-complete**

• **Observations:**
  – a node with \( \text{degree} < n \) \( \Rightarrow \)
    • can always color it successfully, given its neighbors’ colors
  
  – a node with \( \text{degree} = n \) \( \Rightarrow \)
    • can only color if at least two neighbors share same color
  
  – a node with \( \text{degree} > n \) \( \Rightarrow \)
    • maybe, not always
Coloring Algorithm

- **Algorithm:**
  - Iterate until stuck or done
    - Pick any node with degree < n
    - Remove the node and its edges from the graph
  - If done (no nodes left)
    - reverse process and add colors
- **Example (n = 3):**

  ![Graph Example](image)

  - **Note:** degree of a node may drop in iteration
  - Avoids making arbitrary decisions that make coloring fail
More details

- **Apply coloring heuristic**
  
  Build interference graph
  
  Iterate until there are no nodes left
  
  - If there exists a node $v$ with less than $n$ neighbors
    
    push $v$ on register allocation stack
  
  - else
    
    return *(coloring heuristics fail)*
  
  remove $v$ and its edges from graph

- **Assign registers**
  
  While stack is not empty
  
  Pop $v$ from stack
  
  Reinsert $v$ and its edges into the graph
  
  Assign $v$ a color that differs from all its neighbors
What Does Coloring Accomplish?

• **Done:**
  – colorable, also obtained an assignment

• **Stuck:**
  – colorable or not?

```
E ——— A ——— C
  |    |    |
  B    D
```
Extending Coloring: Design Principles

• **A pseudo-register is**
  – Colored successfully: allocated a hardware register
  – Not colored: left in memory

• **Objective function**
  – Cost of an uncolored node:
    • proportional to number of uses/definitions (dynamically)
    • estimate by its loop nesting
  – Objective: **minimize sum of cost of uncolored nodes**

• **Heuristics**
  – Benefit of spilling a pseudo-register:
    • increases colorability of pseudo-registers it interferes with
    • can approximate by its degree in interference graph
  – Greedy heuristic
    • spill the pseudo-register with lowest cost-to-benefit ratio, whenever spilling is necessary
Spilling to Memory

- **CISC architectures**
  - can **operate on data in memory directly**
  - memory operations are **slower than register operations**

- **RISC architectures**
  - machine instructions can **only apply to registers**
  - **Use**
    - must first load data from memory to a register before use
  - **Definition**
    - must first compute RHS in a register
    - store to memory afterwards
  - **Even if spilled to memory, needs a register at time of use/definition**
Chaitin: Coloring and Spilling

• **Identify spilling**
  - Build interference graph
  - Iterate until there are no nodes left
    - If there exists a node \( v \) with less than \( n \) neighbor
      - place \( v \) on stack to register allocate
    - else
      - \( v = \) node with highest degree-to-cost ratio
      - mark \( v \) as spilled
      - remove \( v \) and its edges from graph

• **Spilling may require use of registers; change interference graph**
  - While there is spilling
    - rebuild interference graph and perform step above

• **Assign registers**
  - While stack is not empty
    - Remove \( v \) from stack
    - Reinsert \( v \) and its edges into the graph
    - Assign \( v \) a color that differs from all its neighbors
Spilling

• What should we spill?
  – Something that will eliminate a lot of interference edges
  – Something that is used infrequently
  – Maybe something that is live across a lot of calls?

• One Heuristic:
  – spill cheapest live range (aka “web”)
  – Cost = \([\text{(# defs & uses)} \times 10^{\text{loop-nest-depth}}]/\text{degree}\)
Quality of Chaitin’s Algorithm

• Giving up too quickly

• \(N=2\)

• An optimization: “Prioritize the coloring”
  – Still eliminate a node and its edges from graph
  – Do not commit to “spilling” just yet
  – Try to color again in assignment phase.
Splitting Live Ranges

• **Recall:** Split pseudo-registers into live ranges to create an interference graph that is easier to color
  
  – Eliminate interference in a variable’s “dead” zones.
  – Increase flexibility in allocation:
    • can allocate same variable to different registers

```plaintext
IF A goto L1

B = ... = A
D = B

L1: C = ... = A
    D = = C

A = D

= A
```

```
A

A1

B

D

C

A2
```
Insight

• Split a live range into smaller regions (by paying a small cost) to create an interference graph that is easier to color
  – Eliminate interference in a variable’s “nearly dead” zones.
    • Cost: Memory loads and stores
      – Load and store at boundaries of regions with no activity
    • # active live ranges at a program point can be > # registers

– Can allocate same variable to different registers
  • Cost: Register operations
    – a register copy between regions of different assignments
  • # active live ranges cannot be > # registers
Examples

Example 1:

\[
\begin{align*}
&\text{FOR } i = 0 \text{ TO } 10 \\
&\quad \text{FOR } j = 0 \text{ TO } 10000 \\
&\quad \quad A = A + \ldots \\
&\quad \quad (\text{does not use } B) \\
&\quad \text{FOR } j = 0 \text{ TO } 10000 \\
&\quad \quad B = B + \ldots \\
&\quad \quad (\text{does not use } A)
\end{align*}
\]

Example 2:
Example 1

```
FOR i = 0 TO 10
    FOR j = 0 TO 10000
        A = A + ...  # does not use B
    FOR j = 0 TO 10000
        B = B + ...  # does not use A
```
Example 2
Live Range Splitting

• When do we apply live range splitting?
• Which live range to split?
• Where should the live range be split?
• How to apply live-range splitting with coloring?
  – Advantage of coloring:
    • defers arbitrary assignment decisions until later
  – When coloring fails to proceed, may not need to split live range
    • degree of a node >= n does not mean that the graph definitely is not colorable
  – Interference graph does not capture positions of a live range
One Algorithm

• **Observation**: spilling is absolutely necessary if
  – number of live ranges active at a program point > n

• **Apply live-range splitting before coloring**
  – Identify a point where number of live ranges > n
  – For each live range active around that point:
    • find the outermost “block construct” that does not access
      the variable
  – Choose a live range with the largest inactive region
  – Split the inactive region from the live range
Summary

• **Problems:**
  – Given $n$ registers in a machine, is spilling avoided?
  – Find an assignment for all pseudo-registers, whenever possible.

• **Solution:**
  – **Abstraction:** an *interference graph*
    • nodes: live ranges
    • edges: presence of live range at time of definition
  – **Register Allocation and Assignment** problems
    • equivalent to *n-colorability* of interference graph
      ➔ NP-complete
  – **Heuristics** to find an assignment for $n$ colors
    • **successful:** colorable, and finds assignment
    • **not successful:** colorability unknown & no assignment
CSC D70: Compiler Optimization Register Coalescing

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The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons
Let’s Focus on Copy Instructions

- Optimizations that help optimize away copy instructions:
  - Copy Propagation
  - Dead Code Elimination

- Can all copy instructions be eliminated using this pair of optimizations?
Example Where Copy Propagation Fails

- Use of copy target has multiple (conflicting) reaching definitions

```
X = A + B;
Y = C;
Y = X;
Z = Y + 4;
```
Another Example Where the Copy Instruction Remains

X = A + B;
Y = X;
Z = Y + 4;

C = Y + D;
Y = …;

- Copy target (Y) still live even after some successful copy propagations
- **Bottom line:**
  - copy instructions may still exist when we perform register allocation
Copy Instructions and Register Allocation

• What clever thing might the register allocator do for copy instructions?

• If we can assign both the source and target of the copy to the same register:
  – then we don’t need to perform the copy instruction at all!
  – the copy instruction can be removed from the code
    • even though the optimizer was unable to do this earlier

• One way to do this:
  – treat the copy source and target as the same node in the interference graph
    • then the coloring algorithm will naturally assign them to the same register
  – this is called “coalescing”
Simple Example: Without Coalescing

Without coalescing, X and Y can end up in different registers – cannot eliminate the copy instruction.

```
X = ...;
A = 5;
Y = X;
B = A + 2;
Z = Y + B;
return Z;
```
Example Revisited: With Coalescing

- With coalescing, \(X\) and \(Y\) are now guaranteed to end up in the same register
  - the copy instruction can now be eliminated

- Great! So should we go ahead and do this for every copy instruction?
Should We Coalesce $X$ and $Y$ In This Case?

- It is legal to coalesce $X$ and $Y$ for a \textit{"Y = X"} copy instruction iff:
  - initial definition of $Y$’s live range is this copy instruction, AND
  - the live ranges of $X$ and $Y$ do not interfere otherwise

- But just because it is legal doesn’t mean that it is a good idea...

\[ X = A + B; \]
\[ Y = X; \]
\[ X = 2; \]
\[ Z = Y + X; \]

\textbf{No!} That would result in incorrect behavior if this branch is taken.
Why Coalescing May Be Undesirable

\[ X = A + B; \]
\[ \ldots \ // 100 \text{ instructions} \]
\[ Y = X; \]
\[ \ldots \ // 100 \text{ instructions} \]
\[ Z = Y + 4; \]

• What is the likely impact of coalescing \( X \) and \( Y \) on:
  – live range size(s)?
    • recall our discussion of live range splitting
  – colorability of the interference graph?
• Fundamentally, coalescing adds further constraints to the coloring problem
  – doesn’t make coloring easier; may make it more difficult
• If we coalesce in this case, we may:
  – save a copy instruction, BUT
  – cause significant spilling overhead if we can no longer color the graph
When to Coalesce

• Goal when coalescing is legal:
  – coalesce unless it would make a colorable graph non-colorable

• The bad news:
  – predicting colorability is tricky!
    • it depends on the shape of the graph
    • graph coloring is NP-hard

• Example: assuming 2 registers, should we coalesce X and Y?

![Diagram](image-url)

2-colorable

Not 2-colorable
Representing Coalescing Candidates in the Interference Graph

- To decide whether to coalesce, we augment the interference graph
- Coalescing candidates are represented by a new type of interference graph edge:
  - dotted lines: coalescing candidates
    - try to assign vertices the same color
      - (unless that is problematic, in which case they can be given different colors)
  - solid lines: interference
    - vertices must be assigned different colors

```plaintext
X = ...;
A = 5;
Y = X;
B = A + 2;
Z = Y + B;
return Z;
```
How Do We Know When Coalescing Will Not Cause Spilling?

• **Key insight:**
  – Recall from the coloring algorithm:
    • we can always successfully N-color a node if its degree is < N

• To ensure that coalescing does not cause spilling:
  – check that the degree < N invariant is still locally preserved after coalescing
    • if so, then coalescing won’t cause the graph to become non-colorable
  – no need to inspect the entire interference graph, or do trial-and-error

• **Note:**
  – We do NOT need to determine whether the full graph is colorable or not
  – Just need to check that coalescing does not cause a colorable graph to become non-colorable
Simple and Safe Coalescing Algorithm

- We can safely coalesce nodes $X$ and $Y$ if $(|X| + |Y|) < N$
  - Note: $|X|$ = degree of node $X$ counting interference (not coalescing) edges
- **Example:**

  ![Diagram](image)

  $(|X| + |Y|) = (1 + 2) = 3$

  - if $N \geq 4$, it would always be safe to coalesce these two nodes
    - this cannot cause new spilling that would not have occurred with the original graph
  - if $N < 4$, it is unclear

  *How can we (safely) be more aggressive than this?*
What About This Example?

- Assume $N = 3$
- Is it safe to coalesce $X$ and $Y$?

![Diagram]

$(|X| + |Y|) = (1 + 2) = 3$

(Not less than $N$)

- Notice: $X$ and $Y$ share a common (interference) neighbor: node $A$
  - hence the degree of the coalesced $X/Y$ node is actually 2 (not 3)
  - therefore coalescing $X$ and $Y$ is guaranteed to be safe when $N = 3$
- How can we adjust the algorithm to capture this?
Another Helpful Insight

• Colors are not assigned until nodes are popped off the stack
  – nodes with degree < N are pushed on the stack first
  – when a node is popped off the stack, we know that it can be colored
    • because the number of potentially conflicting neighbors must be < N

• Spilling only occurs if there is no node with degree < N to push on the stack

• Example: (N=2)
Another Helpful Insight

\[ |X| = 5 \]
\[ |Y| = 5 \]

2-colorable after coalescing \( X \) and \( Y \)?
Building on This Insight

• When would coalescing cause the stack pushing (aka “simplification”) to get stuck?
  1. coalesced node must have a degree \( \geq N \)
     • otherwise, it can be pushed on the stack, and we are not stuck
  2. AND it must have at least \( N \) neighbors that each have a degree \( \geq N \)
     • otherwise, all neighbors with degree \( < N \) can be pushed before this node
       – reducing this node’s degree below \( N \) (and therefore we aren’t stuck)

• To coalesce more aggressively (and safely), let’s exploit this second requirement
  – which involves looking at the degree of a coalescing candidate’s neighbors
    • not just the degree of the coalescing candidates themselves
Briggs’s Algorithm

- Nodes $X$ and $Y$ can be coalesced if:
  - $(\text{number of neighbors of } X/Y \text{ with degree } \geq N) < N$

- Works because:
  - all other neighbors can be pushed on the stack before this node,
  - and then its degree is $< N$, so then it can be pushed
  - Example: $(N = 2)$

\[
\begin{array}{c|c|c|c}
X/Y & B & A & Z \\
\end{array}
\]
Briggs’s Algorithm

- Nodes \( X \) and \( Y \) can be coalesced if:
  - \( \text{number of neighbors of } X/Y \text{ with degree } \geq N \) < \( N \)
- **More extreme example:** \( (N = 2) \)
George’s Algorithm

Motivation:
• imagine that \( X \) has a very high degree, but \( Y \) has a much smaller degree
  – (perhaps because \( X \) has a large live range)

• With Briggs’s algorithm, we would inspect all neighbors both \( X \) and \( Y \)
  – but \( X \) has a lot of neighbors!
• Can we get away with just inspecting the neighbors of \( Y \)?
  – showing that coalescing makes coloring no worse than it was given \( X \)?
George’s Algorithm

• Coalescing \( X \) and \( Y \) does no harm if:
  – foreach neighbor \( T \) of \( Y \), either:
    1. degree of \( T \) is \(< N \), or \( \leftarrow \) similar to Briggs: \( T \) will be pushed before \( X/Y \)
    2. \( T \) interferes with \( X \) \( \leftarrow \) hence no change compared with coloring \( X \)

• Example: \((N=2)\)
Summary

• *Coalescing* can enable register allocation to *eliminate copy instructions*
  – if both source and target of copy can be allocated to the same register
• However, coalescing must be applied with care to *avoid causing register spilling*
• Augment the interference graph:
  – dotted lines for coalescing candidate edges
  – try to allocate to same register, unless this may cause spilling
• **Coalescing Algorithms:**
  – simply based upon *degree of coalescing candidate nodes* (**X** and **Y**)
  – Briggs’s algorithm
    • look at degree of neighboring nodes of **X** and **Y**
  – George’s algorithm
    • asymmetrical: *look at neighbors of **Y*** (degree and interference with **X**)

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